IDEATION
CBM for all

The fundamentals of the CBM Predictor algorithm for the enclosure humidity detection

One of the devices that Ideation developed is designed to capture and time stamp undesirable humidity accumulation in IEC 60529 enclosures, covered by the IP code: Degrees of protection provided by enclosures.

Typically, the standard for Ex e / Ex d electrical enclosures (IEC 6079), require traceable waterproofing at all time. Traditionally this is achieved by frequent, time-consuming and costly inspections. Furthermore, inspections add risk of damaging the waterproofing gaskets from opening and closing the enclosures. Repairing a damaged gasket is even more time-consuming. Ideation focus on Abnormal Condition Detection devices could not be more appropriate for such an application.

A general strategy as far as this device is concerned is to use an on-board sophisticated humidity detection algorithm and notify customers to open the cabinet only when absolutely necessary, a substantial saving of effort, time and capital. The device is designed Intrinsic safe to IEC EEx ib and simply penetrates the enclosure via a standard Ex e / Ex d cable gland.

This white paper describes which algorithm is used and what are the merits of this approach.

To deploy humidity algorithm, we borrowed some of the principles from Statistical Process Control (SPC). As the reader might recall, SPC is all about defining the centre line of the process and then establishing the two boundaries, which are the Upper Control Limit (UCL) and Lower Control Limit (LCL).

A general philosophy of a SPC chart is that these three values are defined as:

$$UCL = \mu_x + L\sigma_x \qquad (1)$$

$$\text{Centre line} = \mu_x \qquad (2)$$

$$LCL = \mu_x - L\sigma_x \qquad (3)$$

Where,
μ_x = the mean of x_i
L = distance of the control limits from the centre line in standard deviation units
σ_x = standard deviation of x_i

The above is just a general approach and, depending on application, these equations will take different shapes. However, there are several points that are common to any SPC chart and any specific application. We'll mention the more prominent ones in this white paper.

One of the most elementary points relevant to a variety of SPC charts and applications are the questions of the sample size and the frequency of sampling. In general, larger samples will make it easier to detect small shifts in the process. Equally, if we used smaller samples, but with higher sampling frequency, we detect better the shifts in the process. What is the right mix of the two can be determined either experimentally, or with some assistance from the **average run length** (ARL) calculation.

Essentially, the ARL is the average number of points that must be plotted before a point indicates an out-of-control condition. If the process observations are uncorrelated, then for many common control charts, the ARL can be calculated easily as:

$$ARL = \frac{1}{p} \tag{4}$$

Where,
p = probability that any point exceeds the control limits.

If, for example, *p*=0.0027 (which is equivalent to z=3 and the confidence level of 99.73%), then ARL=370. This can be easily verified using any of the standard SPC tables.

Equation (4) is often referred to as ARL_0 and the other way to interpret the results is to say that on average every 370 points (as in our example above) we will receive a signal that our process is out of control, despite the fact that it is in fact in control.

The same equation (4) is used for calculating the sensitivity of the control chart, although in this case it is called ARL_1. If we are trying to establish ARL_1, then the meaning of *p* is that it is a probability that the process will drift from the target value by certain number of standard deviations. If this probability is *p*=0.2, for example, then ARL_1=5. This means that it will take 5 measurements before we notice that the process is out of control. This is an important feature for the humidity algorithm as some of the control approaches are insensitive to small changes in the process.

To demonstrate how to convert the *p*, the confidence interval and the z-score from ARL, and other way round, we conducted a simple simulation in Excel (see Figure 1).

Column A in Figure 1 has the values of ARL that are given. From there, the level of significance α can be calculated, from which the corresponding confidence level is calculated. Furthermore, using Excel function =NORM.S.INV() in column D, we calculate the z-score for every ARL value.

Column F in Figure 1 shows the reverse process, where we start with the given values of z, from which we calculate the level of significance α using Excel function =NORM.S.DIST(), followed by calculating the corresponding confidence level. For every value of z, we return back to the corresponding ARL values in column I.

Let's take a look at just one of the lines in the spreadsheet, which is the one that refers to our example above. If, for example, ARL=370 (cell A13), then even if process is in control, an out-of-control signal will on average be generated every 370 samples and 99.7% of the time we are going to get the readings as expected (cell C13), which corresponds with the probability of *p*=0.27% (cell B13 shown as 0.0027). This probability is also a chance that a misleading measurement will happen every 370 samples.

	A	B	C	D	E	F	G	H	I
1	From ARL calculate alpha, which leads to conf % and z					From z to alpha, which leads to ARL			
2	ARL	Alpha	Conf %	z		z	Alpha	Conf %	ARL
3	5	0.2	80	1.28155		1.28155	0.2	80	5
4	10	0.1	90	1.64485		1.64485	0.1	90	10
5	22	0.04545	95.4545	2.00042		2	0.0455	95.45	22
6	50	0.02	98	2.32635		2.32635	0.02	98	50
7	100	0.01	99	2.57583		2.57583	0.01	99	100
8	150	0.00667	99.3333	2.71305		2.71305	0.00667	99.3333	150
9	200	0.005	99.5	2.80703		2.80703	0.005	99.5	200
10	250	0.004	99.6	2.87816		2.87816	0.004	99.6	250
11	300	0.00333	99.6667	2.9352		2.9352	0.00333	99.6667	300
12	350	0.00286	99.7143	2.9827		2.9827	0.00286	99.7143	350
13	370	0.0027	99.7297	2.99967		3	0.0027	99.73	370
14	400	0.0025	99.75	3.02334		3.02334	0.0025	99.75	400
15	450	0.00222	99.7778	3.0588		3.0588	0.00222	99.7778	450
16	500	0.002	99.8	3.09023		3.09023	0.002	99.8	500
17	550	0.00182	99.8182	3.11843		3.11843	0.00182	99.8182	550
18	600	0.00167	99.8333	3.14398		3.14398	0.00167	99.8333	600
19	700	0.00143	99.8571	3.18882		3.18882	0.00143	99.8571	700
20	800	0.00125	99.875	3.22722		3.22722	0.00125	99.875	800
21	900	0.00111	99.8889	3.26077		3.26077	0.00111	99.8889	900
22	1000	0.001	99.9	3.29053		3.29053	0.001	99.9	1000
23		=1/A22					=2*(1-NORM.S.DIST(F22,TRUE))		
24			=100*(1-B22)					=100*(1-G22)	
25				=NORM.S.INV(1-(B22/2))					=1/G22

Figure 1. Simulation in Excel to demonstrate relationships between ARL, α, confidence interval and z-scores

To make our simulation even more general, we can show that the relationship between the level of significance α and ARL is inversely exponential in its nature. The graph in Figure 2 illustrates this.

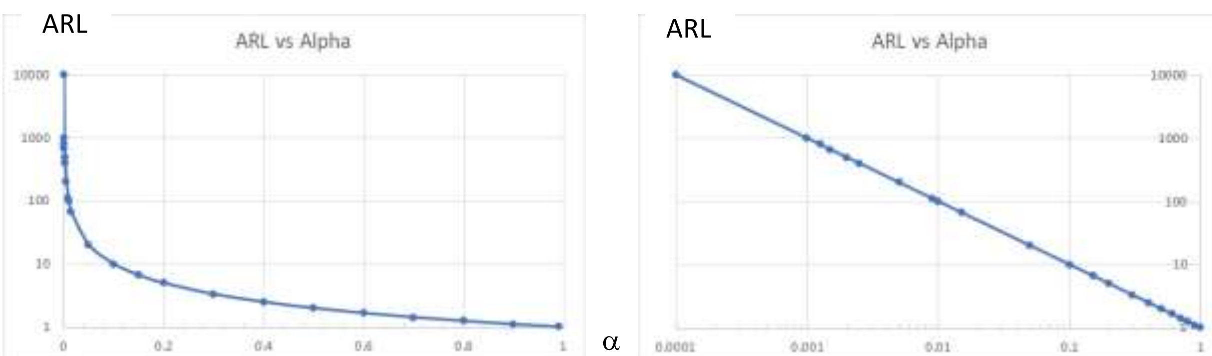

Figure 2. Relationship between ARL and α (left) and the same relationship after the both axes were converted to logarithmic scale (right)

As the graph in Figure 2 on the left shows, for a large value of α (or the corresponding very small confidence interval, if this is easier to use), we will get a small value of ARL. This implies a high-risk strategy as virtually every measurement might be a misleading reading.

We can see that for medium levels of α, say from 0.5 to 0.1 (corresponding to the confidence levels between 50-90%), we can count on every 50th to 90th measurement to be misleading as out of control. Equally, for α somewhere between 0.05 and 0.002 (corresponding confidence 95-99.8%), the misleading out-of-control measurements will appear every 95th to 500th measurement). In relations to this, we will make some recommendations that are very practical at the very end of this white paper.

The graph on the right in Figure 2 shows the same relationship, but both scales are logarithmic, demonstrating a perfect linear relationship.

The second general point we wanted to mention in this white paper is the principle that the control chart boundaries (UCL and LCL) can be understood as the two types of statistical errors. If they are not defined properly, there is a danger of committing one of the two, i.e. either:

Type I error - concluding that the process is out of control, while in effect it is in control

Type II error - concluding that the process is in control, while it is out of control

If we make control limits wide, we decrease the risk of a **type I error** (α), which is the point falling outside the limit and creating a belief that it is out of control, though the process is still in control. The same move will increase the risk of a **type II error** (β), creating an impression that the point is still inside the acceptable range, whilst the process is actually out of control. The opposite is true if we make control limits closer.

The final point that we would like to refer to, and which is common to the majority of the SPC charts and applications, is the questions of the rule when to trigger an alert. One of the possible "inspirations", rather than a set of hard and fast rules, is the Western Electric Handbook that has ever since 1956 remained a standard guidebook. In the Western Electric paper, a set of decision rules for detecting non-random patterns on control charts is suggested. Specifically, it suggests that concluding that the process is out of control if either:

1. One point plots outside the three-sigma control limits,
2. Two out of three consecutive points plot beyond the two-sigma warning limits,
3. Four out of five consecutive points plot at a distance of one-sigma or beyond from the centre line
4. Eight consecutive points plot on one side of the centre line.

Those rules apply to one side of the centre line at a time. Therefore, a point above the upper warning limit followed immediately by a point below the lower warning limit would not signal an out-of-control alarm. These rules were enhanced further by many reputable contributors in this domain (Shewhart is one of them), but for all practical purposes, the above four suffice as a general principle.

A majority of the SPC charts rely on various averages, ranges, standard deviations, etc. to calculate the control corridor. However, with contemporary MEMS technology capturing individual measurements is very easy, so some of the most elementary assumptions of the SPC charts need to be modified. This primarily mean that with individual measurements no rational subgroup is possible and that each individual measurement becomes its own subgroup of size 1. This equally implies that the short-term variability is quantified somewhat differently than with rational subgroups. This leads us to the concept of moving averages as they provide better representation of individual observations and ensure that the short-term variability is handled more appropriately.

Before we come to the actual algorithm, we need to make reference to just one more approach, which is the notion of CUSUM (Cumulative Sum) charts. The CUSUM chart ensures that the information contained in the whole sequence of points is more relevant than just the most recent observation, which is often the case with conventional charts. This means that the sensitivity to smaller changes can be better detected, which is relevant when dealing with humidity. However, we opted for another approach that is very similar to the CUSUM approach, but more robust to cases of autocorrelated values and more immune to the assumption of normality.

A more elegant and more precise approach to controlling the process, such as humidity control, can be achieved effectively by using the Exponentially Weighted Moving Average approach (EWMA). This is the essence of our enclosure humidity detection algorithm.

The EWMA process is defined as:

$$w_t = \lambda x_t + (1 - \lambda) w_{t-1} \quad (5)$$

Where,
x_t = individual measurements
w_t = exponentially weighted moving average values
λ = smoothing constant lambda

The values of lambda are defined as $0 < \lambda \leq 1$ and the values of $w_0 = \mu_0$, or $w_0 = \bar{x}$.

Why is λ between zero and one? Lambda is effectively a correction factor that takes only a fraction of the differences between the actual measurements x_t and the exponentially smoothed values of w_t. If we substitute in equation (5) identical expression for w_{t-1}, and then w_{t-2}, etc., we will realize that the latest value of w_t is actually a function of all the historical measurements x_t. Every historical difference between the actual measurements and the exponentially smoothed values is "discounted" by the value of λ. However, you will also notice that the cumulative value of λ only approaches 1 and in fact shows an exponential drop for every value of λ the further in the past you look. This makes λ a smoothing constant and because it drops exponentially, this contributed towards the name of this method as exponential smoothing approach. In fact, it is combined with moving averages, hence EWMA.

One of the advantages of EWMA approach is that it is as sensitive as CUSUM approach, but at the same time insensitive to nonnormality conditions. This is a very important property and it make this approach very suitable for handling individual signals, such as humidity readings. Besides, humidity measurements exhibit a high degree of autocorrelation, making the EWMA approach ideal to use.

To build the control equations to apply to humidity problem, we need to start with the assumption that if observations x_t are independent random variables with variance σ^2, then the variance of w_t is:

$$\sigma_{w_t}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right) \left[1 - (1 - \lambda)^{2t}\right] \quad (6)$$

From there, we can define the three key equations as:

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1 - \lambda)^{2t}\right]} \quad (7)$$

$$Centre\ line = \mu_0 \quad (8)$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1 - \lambda)^{2t}\right]} \quad (9)$$

Where, L= the width of the control limits (equivalent to z-score that we used earlier in simulation).

As t gets larger, i.e. the further in the past we reach, then $[1-(1-\lambda)^{2t}]$ approaches unity. This means that the control limits equations can be simplified to:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{10}$$

$$Centre\ line = \mu_0 \tag{11}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{12}$$

In fact, in our humidity algorithm we are using both expressions for UCL (LCL is not relevant for humidity applications), as defined by equations (7) and (10). The equation (7) is used to define the set-points during the initialisation phase, and the equations (10) is used during the operations (the rest of the life-cycle) stage. It simplifies the calculations without compromising accuracy.

The algorithm for humidity comes with several recommendations, such as how to define the value of λ and L. We recommend the value of λ as 0.05, 0.1 or 0.2. One has to note that the smaller λ, the smaller changes will be detected (i.e., the system is more sensitive). Regarding the value of L, in general it is recommended that L=3 (the usual 3 sigma), but when $\lambda \leq 0.1$, L can be reduced somewhere between 2.6 to 2.8. If there are specific conditions relevant to this application, the users can experimentally establish which values of λ and L will work for them, though the above guidelines will generally cover most applications.

We can connect the above recommendations with what was said at the beginning of this white paper. For simple control charts, we used ARL_0 as an inverse of the probability that measurements are in control. For EWMA charts, this is a bit more complex. Besides the criterion that defines how many shifts in mean (in terms of multiples of σ), to get a valid answer to this question, we also need the values of L and λ. When λ=0.1 and L=2.7, as one of the recommendations above (which is equivalent to h=5 and k=1/2 from the CUSUM charts, for example), in order to detect the shift of one standard deviation in the process mean, we need only approximately 10 observations. This makes, together with the fact that EWMA is very robust to non-normality, the EWMA approach particularly appropriate to be used for humidity detection.

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