The fundamentals of the CBM Predictor Algorithm for the Process Safety Valves leakage detection

CBM Predictor is an IoT device designed to detect abnormal conditions for process safety valves. The two specific abnormal conditions that CBM Predictor is designed to detect are the valve pop and the potential leak as a consequence of the failure to re-seat after the pop. This white paper will concentrate on the mathematical fundamentals used in the algorithm to detect the leak condition.

The leakage is typically associated with the shift towards ultrasonic frequencies, so the algorithm is based around some advanced engineering techniques known to handle the frequency spectrum successfully. Our objective is to describe how the Fourier transforms are calculated and how is spectral analysis used in the CBM Predictor algorithm.

The starting point are the Fourier series, used to approximate any function as a combination of sine and cosine terms. In fact, the starting point for the algorithm used in CBM Predictor are the Fourier transforms, which are the generalization of the Fourier series. We will skip introduction into Fourier series and focus on the Fourier transforms only.

If you are more familiar with the time window analysis rather than the frequency space analysis, you can think of the power spectrum and the power spectral densities as the Fourier transforms of the autocovariance and the autocorrelation function. It is the same analysis but conducted in a different window, i.e. the frequency window.

If *f(t)* is a periodic function, repeating itself over some interval T, it can be expanded into a complex Fourier series $f_T(t)$ that is expressed as:

$$f_T(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t} \tag{1.1}$$

Where,

c_n = coefficients
n = number of coefficients
t = time
e = the base of the natural logarithms
i = part of the imaginary number, given that $i = \sqrt{-1}$
ω_0 = harmonics of the function, defined as:

$$\omega_0 = \frac{2\pi}{T} \tag{1.2}$$

Unlike the Fourier series, that can handle periodic series only, the Fourier transforms can handle any type of series. From complex analysis, recall the Euler's formula:

$$e^{i\theta} = cos\theta + i\, sin\theta \tag{1.3}$$

Where,

e = the base of the natural logarithms
i = part of the imaginary number, given that $i = \sqrt{-1}$
θ = angle expressed in radians, with the special case where $\theta = 2\pi$, $e^{i2\pi} = 1$

If a signal repeats itself over a period T, then the fundamental frequency f_0 is defined as:

$$f_0 = \frac{1}{T} \tag{1.4}$$

This leads us to the definition of the fundamental angular frequency ω_0, and equation (1.2) can be rewritten as:

$$\omega_0 = 2\pi f_0 \tag{1.5}$$

The units for f_0 are Hz and for ω_0 are radians/second, since there are 2π radians per cycle.

Correspondingly, equation (1.3) can be defined as:

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t) \tag{1.6}$$

Where,

ω = the range of angular frequencies
t = time

From there, a general expression for the Fourier transform is defined as:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \tag{1.7}$$

However, the signal that is being sampled in the CBM Predictor is digital, so the correct version of this transform has to support discrete values. The appropriate statistic to use is a discrete version of the Fourier transform.

The Discrete Fourier Transform (DFT) is defined by the following equation:

$$F(k\Delta f) = \sum_{n=0}^{N-1} f(n\Delta t) e^{-i(2\pi k\Delta f)(n\Delta t)} \tag{1.8}$$

Where,

N = Total number of discrete data points taken
Δt = Time between data points (sampling period)
Δf = Frequency increments (resolution) or frequency bins
k = Number of DFT coefficients, for k=0, 1, 2, ..., N-1
T = Total sampling time

To calculate Δt, we need:

$$\Delta t = \frac{T}{N}, \qquad (1.9)$$

And, to calculate Δf, we need:

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta t} \qquad (1.10)$$

From the above, the sampling frequency f_s is calculated as:

$$f_s = \frac{1}{\Delta t} = \frac{N}{T} \qquad (1.11)$$

This also provides and alternative equation for Δf (1.10) as:

$$\Delta f = \frac{f_s}{N} \qquad (1.12)$$

For implementation reasons, it is customary (though not necessary) to calculate N^2 number of DFTs, which usually means 256, 512, 1024, 2048, etc. number of DFTs.

According to the Nyquist criterion, the maximum number of DFTs that should be calculated should not exceed $f_s/2$. The value of k, for which the frequency kΔf equals $f_s/2$, is calculated as shown below.

We define kΔf as:

$$k\Delta f = \frac{f_s}{2} \qquad (1.13)$$

From (1.13), we can extract k as:

$$k = \frac{f_s}{2\Delta f} = \frac{\frac{N}{T}}{2\frac{1}{T}} = \frac{N}{2} \qquad (1.14)$$

From there it follows that the maximum useful frequency (or, folding frequency f_f) for which it makes sense to calculate the DFT is:

$$f_f = \frac{f_s}{2} = \frac{N}{2}\Delta f \qquad (1.15)$$

In the case of the CBM Predictor algorithm, we use an alternative version of equation (1.8):

$$F_k = \sum_{n=0}^{N-1} x_n \, e^{\frac{-i2\pi kn}{N}} \qquad (1.16)$$

Where, all the symbols remain the same and x_n is the input signal.

As the formula (1.16) indicates, F_k are complex numbers. A complex number of the form: $z = a + bi$, has a real and imaginary part. The magnitude of the complex number is calculated as the modulus, which is $|z| = \sqrt{a^2 + b^2}$. This means that the magnitude of the complex number is equivalent to the amplitude of the DFT. We use expressions magnitude and amplitude interchangeably.

Generically, we can present a DFT, or F_k, as a complex number, in the following simplified way:

$$F_k = A_k + iB_k \tag{1.17}$$

To calculate the amplitude, or magnitude, for F_k in (1.17), we effectively need to use the formula:

$$G_k = \sqrt{A_k^2 + B_k^2} \tag{1.18}$$

Although the equation (1.18) provides the value of the magnitude (or amplitude) of the DFT, to complete the analysis we need to normalize it for the size of the frequency bin. The normalized version of the DFT amplitude yields the Power Spectral Density (PSD) function. However, the normalization of the Fourier transforms F_k has two separate cases, depending on the value of k.

To calculate properly normalized magnitudes G_k, we differentiate the case for k=0 and all other for k=1, 2, …, ($\frac{N}{2}$-1). For k=0, the magnitude is calculated as:

$$G_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n \tag{1.19}$$

For k=1, 2, …, ($\frac{N}{2}$-1), the magnitudes are:

$$G_k = \frac{2}{N}\sum_{n=0}^{N-1} x_n \, e^{\frac{-i2\pi nk}{N}} \tag{1.20}$$

As we can see equation (1.19) is a truncated version of (1.16), with just 1/N multiplier that implies the average (or normalization), because at this frequency F_0 is at full amplitude. Equation (1.20) uses number 2 in the numerator of the multiplier, because this converts the half amplitude into the full amplitude, making all the remaining F_k comparable to F_0.

To summarise, to complete our calculations, we need to find the modulus value of the complex number F_k and normalize it (divide by N) and multiplied by the appropriate factor to get the full amplitude value.

Effectively, the equations (1.19) and (1.20) can be rewritten as:

For k=0 $$G_k = \frac{|F_k|}{N} \tag{1.21}$$

For k>0 $$G_k = \frac{2|F_k|}{N} \tag{1.22}$$

The Fourier power is calculated as:

For k=0 $$PSD_0(F_k) = \frac{G_k^2}{2} \quad (1.23)$$

For k>0 $$PSD_x(F_k) = G_k^2 \quad (1.24)$$

Expression (1.23) and (1.24), or the power spectral densities, are also informally called the power spectrum. Sometimes, the power spectrum is specified as:

$$PSD_x(f_k) = \frac{F_k F_k^*}{N^2} \quad (1.25)$$

Where, F_k^* is a complex conjugate of F_k, which means that the sign of the imaginary part of F_k is reversed. Remember that if you multiply a complex number with its conjugate, you always get the same outcome, e.g. (a+bi) × (a-bi) = $a^2 + b^2$, which takes us back to our equation (1.18).

As we indicated at the beginning, the DFTs transform the vibration signal into amplitudes as a function of frequency. A spectrogram, on the other hand, presents the signal in a 3D space, where the coordinates include the time as x coordinate, amplitude as y coordinate and frequency as z coordinate. The tool used in the CBM Predictor algorithm is another technique, a close relative of the previous two, called the power spectral density. It takes amplitudes from the DFTs and normalizes them to the frequency bin width (i.e. to the resolution of the frequency increments). Effectively, we calculate how much power is contained in every frequency and how these powers are distributed per every frequency, hence the phrase power spectral density.

Textbook examples typically use a snapshot of the signal to demonstrate how the DFTs and PSDs are calculated. In our algorithm, the DFTs and PSDs are calculated in real time, which means that approximately every 30 milli seconds we calculate the DFT of the signal coming from the process safety valve and calculate the amplitudes for every DFT in the sequence (1024 or 2048 of them, depending on application). This is over 30 real-time DFT calculations every second.

Once these DFTs are extracted, the PSDs are calculated and compared against certain criteria, which ultimately provides an indication of the leakage, or otherwise.

The sampling is conducted at 96 kHz to ensure that DFTs capture all the frequencies up to 48,000 Hz. The leakage is usually associated with the ultrasonic frequency range and, although the humans cannot hear anything above 20,000 Hz, the CBM Predictor can capture the whole ultrasonic range up to 48 kHz.

In summary, IDEATION's CBM Predictor is primarily an abnormal condition detector operating on a standard IoT network. It is the foundation for the condition-based maintenance. In essence, the CBM Predictor has a direct and positive impact on the reliability of the process unit, and subsequently the whole plant. IDEATION believes in simple products and simple processes. The predictive maintenance strategies that our solutions support are currently the best available approach to maximise the uptime, reduce overall maintenance costs and ensure robust health, safety and environmental compliance.